

Dark energy via multi-Higgs doublet models: accelerated expansion of the Universe in inert doublet model scenario

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Abstract

Scalar fields are among the possible candidates for dark energy. This paper is devoted to the scalar fields from the inert doublet model, where instead of one as in the standard model, two SU(2) Higgs doublets are used. The component fields of one SU(2) doublet (ϕ_1) act in an identical way to the standard model Higgs while the component fields of the second SU(2) doublet (ϕ_2) are taken to be the dark energy candidate (which is done by assuming that the phase transition in the field has not yet occurred). It is found that one can arrange for late time acceleration (dark energy) by using an SU(2) Higgs doublet in the inert Higgs doublet model, whose vacuum expectation value is zero, in the quintessential regime.

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I. INTRODUCTION

There are at least three components which contribute to the total energy density of the Universe. These are non-relativistic matter, relativistic matter and dark energy. Dark energy, which constitutes about 70% of the present energy density of the Universe is one of the most discussed and hot topics of this era. Several possible candidates (explanations) of inflation and dark energy are discussed in the literature, a few of them are: the cosmological constant (Λ), for a brief review see [1, 2]; modified gravity, for a good discussion see [3, 4]; scalar field models (e.g. quintessence, K-essence, tachyon field, phantom (ghost) field [5–7], dilatonic dark energy, Chaplygin gas) [2] and vector fields[8] [9].

Scalar, vector and tensorial fields came into existence with the development of quantum field theory and were first used by Alan Guth [10], Andre Linde [11] (plus references therein for discussion), to propose inflation as a possible solution to the horizon and flatness problems. For a review on inflation see [12].

The homogeneous, isotropic Universe model is described by the Friedmann-Robertson-Walker (FRW) metric and its dynamics is described by the Friedmann equations which are

$$H^2 = \frac{1}{3}\rho - \frac{\kappa}{a^2} , \quad (1)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(1 + 3\omega_{\text{eff}})\rho , \quad (2)$$

where a is the scale factor, $H = \dot{a}/a$ is the Hubble parameter, ρ is the total energy density, κ is spatial curvature and P is the pressure. Where, $\omega_{\text{eff}} = \Omega_{DE} \omega_{DE} + \Omega_R \omega_R + \Omega_M \omega_M$ and the barotropic equation of state $P = \omega\rho$ has been used. In the equations, natural units $\hbar = c = 1$ and $(8\pi G)^{-1/2} = M_P = 1$ have been in above and subsequent equations. From eq. (2), we see that $\ddot{a} > 0$ when $\omega_{\text{eff}} < -\frac{1}{3}$.

Using the law of conservation of energy, it can easily be derived that

$$\rho \propto a^{-3(1+\omega)} . \quad (3)$$

From eq. (3) scalar fields are categorized into two categories on the basis of their equation of state (EoS) parameter ω :

The Quintessence field: These fields lie in the domain $-1 < \omega < -1/3$. The main feature of this field is that its energy density decrease as in relativistic ($\omega = 1/3$) or non-relativistic matter ($\omega = 0$), but the field could be a possible candidate for dark energy since $\omega < -1/3$ [2].

The Phantom field: These fields lie in the domain $\omega < -1$ and have the property that their energy density increases as the Universe expands and the scale factor ‘ a ’ becomes infinite in a finite time. These type of fields have negative kinetic energy, which allows ω to take value less than -1 [2].

When $\omega = -1$ the energy density does not change as the Universe expands and remains constant. The deviation of ω from -1 tells how us energy density, ρ , of the field changes as the Universe expands.

The scalar fields used to explain accelerated expansion of the Universe are taken to be new fields (with no connection with Particle Physics). However, the introduction of new fields with no experimental basis other than “explaining” one observed phenomenon is too reminiscent of epicycles or the per-relativistic aether. Only if one can exclude fields contained in the standard model, or a minor extension of it, would it be justifiable to introduce such exotic proposals as phantom or quintessence fields, which I argue is necessary and essential to made a unified model of matter and energy.

In this paper, I assume that the present state of the Universe is described by an inert Higgs doublet model. If the phase transition in the second SU(2) doublet, ϕ_2 , (which has zero vacuum expectation value (vev)) has not occurred yet, then the component fields of the doublet can be considered as dark energy candidates. In the case when the present Universe is described by an inert Higgs doublet model after the second phase transitions different forces and particles would arise (depending upon the VeV of the second doublet) which can be avoided by taking the parameter m_{22}^2 of Higgs potential zero thus second phase transition never occurs. The results obtained are presented which favor that accelerated expansion of the Universe is quite possible (in quintessence regime)[13]. Previously, E. Greenwood et al in [14] considered the appearance of a second vacuum in the SM Higgs potential. They showed that if the second order electroweak phase transition is followed by a first order phase

transition (which they were able to achieve for a certain range of parameters of their model) then this phase transition can cause the late time acceleration of the Universe. Carroll et al have also given an independent phantom field model in [5] where they have also discussed the stability of energy density along with phantom field as a possible dark energy candidate. V. K. Onemli and R. P. Woodard have also showed in [6, 7] that Quantum effects in Cosmology could cause the violation of the null and weak energy conditions (i.e. $\rho + P < 0$ in their model) causing $\omega < -1$ on cosmological scales without introducing any ghost, phantom, etc..

The paper is organized as follows; in section 2 we review the inert doublet model and obtain constraints on the inert doublet model's potential to be positive along with the calculations of the minima of the potential. In the next section phase transitions of inert doublet model has been discussed. Section 4 is devoted to obtaining the answer to the question; whether Higgs field can be a possible candidate for dark energy in inert doublet model? In section 5, we discuss the decays of Higgs field(s). Section 6 is where we conclude the paper with future work possibilities.

II. THE INERT DOUBLET MODEL

The electroweak symmetry in the standard model (SM) of Particle Physics is broken spontaneously by the non-zero vev of the Higgs field(s) via Higgs mechanism. The Lagrangian which describes any model in Particle Physics is

$$\mathcal{L} = \mathcal{L}_{gf}^{SM} + \mathcal{L}_Y + \mathcal{L}_{Higgs} . \quad (4)$$

Here, \mathcal{L}_{gf}^{SM} is the $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ is the SM interaction of the fermions and gauge bosons (force carriers) [15] , \mathcal{L}_Y is the Yukawa interaction of fermions with the Higgs field(s)[16] and \mathcal{L}_{Higgs} is the Higgs field Lagrangian where

$$\mathcal{L}_{Higgs} = T_H - V_H , \quad (5)$$

T_H being the kinetic term of the Higgs field(s) and V_H being potential of the Higgs field(s)[17]. Here,

$$T_H = (D_{1\mu}\phi_1)^\dagger(D_1^\mu\phi_1) + (D_{2\mu}\phi_2)^\dagger(D_2^\mu\phi_2) + [\chi(D_{1\mu}\phi_1)^\dagger(D_2^\mu\phi_2) + \chi^*(D_{2\mu}\phi_2)^\dagger(D_1^\mu\phi_1)] , \quad (6)$$

and

$$V_H = V_1 + V_2 + V_{int} , \quad (7)$$

$$\begin{aligned} &= \rho_1 \exp(\Lambda_1 m_{11}^2 \phi_1^\dagger \phi_1) + \rho_2 \exp(\Lambda_2 m_{22}^2 \phi_2^\dagger \phi_2) + \rho_3 \exp\left(\frac{1}{2} \Lambda_3 \lambda_1 (\phi_1^\dagger \phi_1)^2\right) \\ &+ \rho_4 \exp\left(\frac{1}{2} \Lambda_4 \lambda_2 (\phi_2^\dagger \phi_2)^2\right) + m_{12}^2 (\phi_1^\dagger \phi_2) + m_{12}^{2*} (\phi_2^\dagger \phi_1) + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) \\ &+ \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{1}{2} [\lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_5^* (\phi_2^\dagger \phi_1)^2] + \lambda_6 (\phi_1^\dagger \phi_1) (\phi_1^\dagger \phi_2) \\ &+ \lambda_6^* (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_1) + \lambda_7 (\phi_2^\dagger \phi_2) (\phi_1^\dagger \phi_2) + \lambda_7^* (\phi_2^\dagger \phi_2) (\phi_2^\dagger \phi_1) , \end{aligned} \quad (8)$$

where V_1 and V_2 in eq. (7) are the Lagrangian of Higgs field ϕ_1 (given by first two terms of RHS of eq. (8)) and ϕ_2 (given by 3rd and 4th terms of RHS of eq. (8)) respectively, V_{int} is the interaction Lagrangian of fields ϕ_1 and ϕ_2 , given by remaining terms,

$$\begin{aligned} D_{1\mu} &= \partial_\mu + i \frac{g_1}{2} \sigma_i W_\mu^i + i \frac{g_1'}{2} B_\mu , \\ D_{2\mu} &= \partial_\mu + i \frac{g_2}{2} \sigma_i W_\mu^i + i \frac{g_2'}{2} B_\mu , \\ \phi_i &= \begin{bmatrix} \phi_i^+ \\ \eta_i + i\chi_i + \nu_i \end{bmatrix} , \quad \phi_i^\dagger = \begin{bmatrix} \phi_i^- & \eta_i - i\chi_i + \nu_i \end{bmatrix} . \end{aligned}$$

The dimensions of different parameters are

$$[\rho_i]^{-1} = [\Lambda_i] = [L]^4, \quad [m_{ii}^2] = [L]^{-2}, \quad [\phi_i] = [L]^{-1} \text{ and } [\lambda_i] = [L]^0.$$

Here “ L ” denotes the length. The fields ϕ_i^+ , ϕ_i^- , η_i and χ_i are the hermitian Higgs fields (ϕ_i^\pm are charged whereas other fields are neutral), ν_i is the vev of the doublet ϕ_i .

The Higgs vacuum energy of the potential given by eq. (8) is

$$\begin{aligned} E_{vac} &= \rho_1 \exp\left(\frac{1}{2} m_{11}^2 \Lambda_1 \nu_1^2\right) + \rho_2 \exp\left(\frac{1}{2} m_{22}^2 \Lambda_2 \nu_2^2\right) + \rho_3 \exp\left(\frac{1}{8} \lambda_1 \Lambda_3 \nu_1^4\right) \\ &+ \rho_4 \exp\left(\frac{1}{8} \lambda_2 \Lambda_4 \nu_2^4\right) + \frac{1}{4} \lambda_3 \nu_1^2 \nu_2^2 + \frac{1}{4} \lambda_4 \nu_1^2 \nu_2^2 + \frac{1}{8} (\lambda_5 + \lambda_5^*) \nu_1^2 \nu_2^2 \\ &+ \frac{1}{4} (\lambda_6 + \lambda_6^*) \nu_1^3 \nu_2 + \frac{1}{4} (\lambda_7 + \lambda_7^*) \nu_1 \nu_2^3 . \end{aligned} \quad (9)$$

We would be choosing $\nu_2 = 0$ here (explained in section II B), which corresponds to the inert doublet model, then

$$E'_{vac} = \rho_2 + \rho_4 + \rho_1 \exp\left(\frac{1}{2}m_{11}^2\Lambda_1\nu_1^2\right) + \rho_3 \exp\left(\frac{1}{8}\lambda_1\Lambda_3\nu_1^4\right). \quad (10)$$

Since we are looking to provide the accelerated expansion to the Universe from Higgs field(s) we want the Higgs field(s) to live longer than the age of the Universe (or at least not less than the present age of the Universe). Thus, stable Higgs field(s) is(are) required for the scalar field dark energy model. This can be achieved by imposing a discrete Z_2 symmetry $\phi \rightarrow -\phi$. When Z_2 symmetry is broken by $(\phi_i^\dagger\phi_j)$ type terms then it is said to be softly broken and when Z_2 symmetry is broken by $(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_l)$ type terms then symmetry is said to be hardly broken. The terms containing m_{12}^2 describe the soft symmetry breaking of Z_2 symmetry, the terms λ_6 and λ_7 describe the hard symmetry breaking of Z_2 symmetry. In the absence of these terms along with no cross kinetic term i.e. $\chi = 0$, the 2HDM's Higgs Lagrangian has a perfect Z_2 symmetry [18]. There are two Z_2 symmetries corresponding to the doublets:

$$\text{I:} \quad \phi_1 \longrightarrow -\phi_1, \quad \phi_2 \longrightarrow \phi_2, \quad (11)$$

$$\text{II:} \quad \phi_1 \longrightarrow \phi_1, \quad \phi_2 \longrightarrow -\phi_2. \quad (12)$$

For Z_2 symmetry see [18].

It should also be noted that even if the Higgs field(s) has(have) decayed off its vacuum energy given by eq. (9) would still be able to provide accelerated expansion provided it dominates at some time in the life span of the Universe.

A. Positivity constraint on Higgs potential

The stability of the vacuum given by eq. (8) implies that the potential must be positive for all asymptotically large values of the Higgs field. Even though V_1 and V_2 remains positive for all values of fields considering $\rho_i > 0$, we should find out the constraints on the parametric product $\Lambda_1 m_{11}^2$, $\Lambda_2 m_{22}^2$, $\frac{1}{2}\Lambda_3\lambda_1$ and $\frac{1}{2}\Lambda_4\lambda_2$ when the exponentials are approximated upto a finite order (4th here). The conditions obtained should match on to the conditions for the general 2HDM potential given in [18].

Some conditions can be obtained to achieve the stability of the vacuum by introducing new parameters as [19, 20],

$$|\phi_1| = r \cos \gamma, \quad |\phi_2| = r \sin \gamma, \quad \frac{\phi_2^\dagger \phi_1}{|\phi_1| |\phi_2|} = \rho \exp(i\theta),$$

where $\gamma \in [0, \pi/2]$, $\rho \in [0, 2\pi]$. Using these transformations in the potential within its expanded form upto forth order (with $\rho_i > 0$ and hence will be omitted in eq. (13)), the potential becomes

$$V = r^4 \left(\lambda'_1 \cos^4 \gamma + \lambda'_2 \sin^4 \gamma + \lambda_3 \cos^2 \gamma \sin^2 \gamma + \lambda_4 \rho^2 \cos^2 \gamma \sin^2 \gamma + \lambda_5 \rho^2 \cos^2 \gamma \sin^2 \gamma \cos 2\theta + 2\rho \cos \gamma \sin \gamma \cos \theta [\lambda_6 \cos^2 \gamma + \lambda_7 \sin^2 \gamma] \right), \quad (13)$$

the positivity of eq. (13) implies that

$$\lambda'_1 > 0, \quad \lambda'_2 > 0, \quad \lambda_3 > -2 \sqrt{\lambda'_1 \lambda'_2}, \quad (14)$$

when $\lambda_6 = \lambda_7 = 0$, then we also have

$$\lambda_3 + \min[0, \lambda_4 - |\lambda_5|] > -2 \sqrt{\lambda'_1 \lambda'_2}, \quad (15)$$

where $\lambda'_1 = \frac{1}{2} (\rho_3 \Lambda_3 \lambda_1 + \rho_1 (\Lambda_1 m_{11}^2)^2)$ and $\lambda'_2 = \frac{1}{2} (\rho_4 \Lambda_4 \lambda_2 + \rho_2 (\Lambda_2 m_{22}^2)^2)$.

B. Minimizing the Higgs potential

The extrema of the potential are found by taking

$$\left. \frac{\partial V_H}{\partial \phi_1} \right|_{\substack{\phi_1 = \langle \phi_1 \rangle \\ \phi_2 = \langle \phi_2 \rangle}} = \left. \frac{\partial V_H}{\partial \phi_1^\dagger} \right|_{\substack{\phi_1 = \langle \phi_1 \rangle \\ \phi_2 = \langle \phi_2 \rangle}} = 0 \quad \text{and} \quad \left. \frac{\partial V_H}{\partial \phi_2} \right|_{\substack{\phi_1 = \langle \phi_1 \rangle \\ \phi_2 = \langle \phi_2 \rangle}} = \left. \frac{\partial V_H}{\partial \phi_2^\dagger} \right|_{\substack{\phi_1 = \langle \phi_1 \rangle \\ \phi_2 = \langle \phi_2 \rangle}} = 0. \quad (16)$$

The most general solution of the conditions (16) is

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu_1 \end{pmatrix} \quad \text{and} \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ \nu_2 \end{pmatrix}.$$

The first solution of extrema has been taken to be similar to the Higgs vacuum in SM and the second one is the most general which can occur. One needs to keep in mind that now $\nu^2 = \nu_1^2 + |\nu_2|^2 + u^2$. In the electroweak symmetry breaking situation where $\langle \phi_1 \rangle \neq 0$, choosing the z-axis in weak isospin space such that $\nu_1 \geq 0$.

When $u \neq 0$, the non-zero value of u will contribute to the “charged” type dark energy, which has not been observed. To avoid this, we would take $u = 0$. From the extrema conditions given by eq. (16), we can determine the values of ν_1 and ν_2 [19, 21], solving eq. (16) for the potential given by eq. (8) leads to

$$\begin{aligned} \nu_1 \left[2\rho_1\Lambda_1 m_{11}^2 \exp\left(\frac{\Lambda_1 m_{11}^2}{2}\nu_1^2\right) + \rho_3\Lambda_3\lambda_1\nu_1^2 \exp\left(\frac{\Lambda_3\lambda_1}{8}\nu_1^4\right) - 2\text{Re}(m_{12}^2)\frac{\nu_2}{\nu_1} \right. \\ \left. + (\lambda_3 + \lambda_4 + \text{Re}(\lambda_5))\nu_2^2 + 3\text{Re}(\lambda_6)\nu_1\nu_2 + \text{Re}(\lambda_7)\frac{\nu_2^3}{\nu_1} \right] = 0 , \end{aligned}$$

$$\begin{aligned} \nu_2 \left[2\rho_2\Lambda_2 m_{22}^2 \exp\left(\frac{\Lambda_2 m_{22}^2}{2}\nu_2^2\right) + \rho_4\Lambda_4\lambda_2\nu_2^2 \exp\left(\frac{\Lambda_4\lambda_2}{8}\nu_2^4\right) - 2\text{Re}(m_{12}^2)\frac{\nu_1}{\nu_2} \right. \\ \left. + (\lambda_3 + \lambda_4 + \text{Re}(\lambda_5))\nu_1^2 + \text{Re}(\lambda_6)\frac{\nu_1^3}{\nu_2} + 3\text{Re}(\lambda_7)\nu_1\nu_2 \right] = 0 . \end{aligned}$$

The solution of the above equations for ν_1 and ν_2 is practically impossible since the exponentials contain ν_1 and ν_2 . The vev’s are approximated by truncating the potential eq. (8) upto forth order. This is also a good approximation in the sense that the results of ν ’s would also be true for the general 2HDM potential vev’s of [18]. Applying minimization conditions given by eq. (16) we get,

$$\begin{aligned} \nu_1 \left[2\rho_1\Lambda_1 m_{11}^2 + 2\lambda'_1\nu_1^2 - 2\text{Re}(m_{12}^2)\frac{\nu_2}{\nu_1} + (\lambda_3 + \lambda_4 + \text{Re}(\lambda_5))\nu_2^2 \right. \\ \left. + 3\text{Re}(\lambda_6)\nu_1\nu_2 + \text{Re}(\lambda_7)\frac{\nu_2^3}{\nu_1} \right] = 0 , \end{aligned} \quad (17)$$

$$\begin{aligned} \nu_2 \left[2\rho_2\Lambda_2 m_{22}^2 + 2\lambda'_2\nu_2^2 - 2\text{Re}(m_{12}^2)\frac{\nu_1}{\nu_2} + (\lambda_3 + \lambda_4 + \text{Re}(\lambda_5))\nu_1^2 \right. \\ \left. + \text{Re}(\lambda_6)\frac{\nu_1^3}{\nu_2} + 3\text{Re}(\lambda_7)\nu_1\nu_2 \right] = 0 , \end{aligned} \quad (18)$$

where $\lambda'_1 = \frac{1}{2}(\rho_3\Lambda_3\lambda_1 + \rho_1(\Lambda_1 m_{11}^2)^2)$ and $\lambda'_2 = \frac{1}{2}(\rho_4\Lambda_4\lambda_2 + \rho_2(\Lambda_2 m_{22}^2)^2)$ as given before.

Since Z_2 symmetry is very important in determining the phenomenology of the theory and with exact Z_2 symmetry the lightest Higgs boson (field) is stable. In this article, the potential which is Z_2 symmetric will also be taken and the possibility of any Higgs field(s) to be the candidate for the dark energy will be examined, if possible it would give an ever expanding Universe while accelerating. Thus, imposing the Z_2 symmetry [18] we have,

$$\chi = 0 , \quad m_{12}^2 = 0 , \quad \lambda_6 = 0 , \quad \lambda_7 = 0 , \quad (19)$$

Using eqs. (17, 18, 19), we get four solutions for ν_1 and ν_2 , which are

$$\nu_1^2 = 0, \quad \nu_2^2 = 0, \quad (20)$$

$$\nu_1^2 = 0, \quad \nu_2^2 = -\frac{\rho_2 \Lambda_2 m_{22}^2}{\lambda_2'}, \quad (21)$$

$$\nu_1^2 = -\frac{\rho_1 \Lambda_1 m_{11}^2}{\lambda_1'}, \quad \nu_2^2 = 0, \quad (22)$$

$$\nu_1^2 = -\frac{2(2\rho_1 \Lambda_1 \lambda_2' m_{11}^2 - \rho_2 \Lambda_2 \lambda_{345} m_{22}^2)}{4\lambda_1' \lambda_2' - \lambda_{345}^2}, \quad \nu_2^2 = -\frac{2(2\rho_2 \Lambda_2 \lambda_1' m_{22}^2 - \rho_1 \Lambda_1 \lambda_{345} m_{11}^2)}{4\lambda_1' \lambda_2' - \lambda_{345}^2}, \quad (23)$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \text{Re}(\lambda_5)$.

Electroweak symmetric vacuum This vacuum solution is given by eq. (20) represents the electroweak symmetric extremum, in this case the complete Lagrangian respects Z_2 symmetry with respect to both fields given by eq. (11) and eq. (12) i.e. I and II, here $m_{11}^2 > 0$ and $m_{22}^2 > 0$. The doublets are

$$\phi_i = \begin{bmatrix} \phi_i^+ \\ \eta_i + i\chi_i \end{bmatrix} \quad \text{and} \quad \phi_i^\dagger = \begin{bmatrix} \phi_i^- & \eta_i - i\chi_i \end{bmatrix}.$$

Inert vacuum-I Eq. (21) will be called inert vacuum-I. The doublets in this vacuum are

$$\phi_1 = \begin{bmatrix} \phi_1^+ \\ \eta_1 + i\chi_1 \end{bmatrix} \quad \text{and} \quad \phi_2 = \begin{bmatrix} \phi_2^+ \\ \eta_2 + i\chi_2 + \nu_2 \end{bmatrix},$$

in this type of vacuum solution, the fields ϕ_2^\pm and χ_2 are the Goldstone bosons and all other fields are physical. In this case, the complete Lagrangian respects Z_2 symmetry given by eq. (11) and only the Higgs Lagrangian (not the complete Lagrangian because of the Yukawa Lagrangian since ϕ_2 is not contained in Yukawa Lagrangian) violates given by eq. (12).

Inert vacuum-II The doublets in this type of vacuum, given by eq. (22), are

$$\phi_1 = \begin{bmatrix} \phi_1^+ \\ \eta_1 + i\chi_1 + \nu_1 \end{bmatrix}, \quad \phi_2 = \begin{bmatrix} \phi_2^+ \\ \eta_2 + i\chi_2 \end{bmatrix}, \quad (24)$$

the fields ϕ_1^\pm and χ_1 are the Goldstone bosons whereas other fields are physical. The Yukawa interaction is described by the interaction of field ϕ_1 with fermions (ϕ_2 does not couple to fermions but it can appear in loop process).

The Higgs and Yukawa Lagrangian violates Z_2 symmetry given by eq. (11) whereas respects given by eq. (12). Thus, parity in field ϕ_2 is conserved this makes the lightest particle/field of doublet 2 stable. If there is no phase transitions (which brings Universe in the mixed vacuum state) in the future of the Universe or before the phase transitions in doublet ϕ_2 then this lightest particle is a good candidate for dark energy, giving rise to an ever accelerating expanding Universe. Without the stability, we would have a model for accelerated expanding Universe for a limited time depending upon the decay width (life time) of the dark energy field(s), because if field(s) can decay then it ceases to provide accelerated expansion after its decay to the other fields only if its vacuum energy has not dominates before its decay.

The masses of the fields in this vacuum are

$$\begin{aligned} m_{\eta_1}^2 &= 2\lambda_1' \nu_1^2, \\ m_{\eta_2}^2 &= \rho_2 \Lambda_2 m_{22}^2 + \frac{\lambda_3 + \lambda_4 + \text{Re}(\lambda_5)}{2} \nu_1^2, \\ m_{\chi_2}^2 &= \rho_2 \Lambda_2 m_{22}^2 + \frac{\lambda_3 + \lambda_4 - \text{Re}(\lambda_5)}{2} \nu_1^2, \\ m_{\phi_2^\pm}^2 &= \rho_2 \Lambda_2 m_{22}^2 + \frac{\lambda_3}{2} \nu_1^2, \end{aligned} \tag{25}$$

where $\nu_1 = \frac{1}{\sqrt[4]{2G_F^2}} \approx 246\text{GeV}$.

In this paper, I am interested in this type of vacuum.

Mixed vacuum After symmetry is broken, the Lagrangian violates the Z_2 symmetry given by eqs. (11) and (12). The physical fields in this model are the combination of fields from ϕ_1 and ϕ_2 . The complete theory and phenomenology of this model is discussed in [22]. Mixed vacuum solution is given by eq. (23).

III. PHASE TRANSITIONS IN INERT DOUBLET MODEL/TWO HIGGS DOUBLET MODEL

When we look for dark energy to be some physical field(s), then it becomes essential to also look for its evolution in the history (cooling) of the Universe.

According to [21], in the quantum field theory at non-zero temperature the terms m_{ii}^2 of

the quadratic terms evolve with temperature as

$$m_{11}^2 \longrightarrow m_{11}^2 + \frac{1}{2}c_1 T^2, \quad (26)$$

$$m_{22}^2 \longrightarrow m_{22}^2 + \frac{1}{2}c_2 T^2, \quad (27)$$

where

$$c_1 = \frac{6\lambda'_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g_1^2 + g_1'^2}{32} + \frac{g_t^2 + g_b^2}{8},$$

and

$$c_2 = \frac{6\lambda'_2 + 2\lambda_3 + \lambda_4}{12} + \frac{3g_2^2 + g_2'^2}{32},$$

here $\lambda'_1 = \frac{1}{2}(\rho_3\Lambda_3\lambda_1 + \rho_1(\Lambda_1 m_{11}^2)^2)$, $\lambda'_2 = \frac{1}{2}(\rho_4\Lambda_4\lambda_2 + \rho_2(\Lambda_2 m_{22}^2)^2)$ as above and $g_{1,2}$ and $g'_{1,2}$ are the Electroweak gauge couplings with doublet ϕ_1 and ϕ_2 respectively, $g_t \approx 1$ and $g_b \approx 0.03$ are the top and bottom quark Yukawa couplings with the doublet ϕ_1 in inert doublet model respectively.

In general, c_1 and c_2 can have any sign but the potential positivity implies that (in any situation)

$$c_1 + c_2 > 0,$$

with the above mentioned positivity constraints, the case when $c_1 > 0$ and $c_2 > 0$ will be considered.

The Higgs potential with the new quadratic terms now become

$$\begin{aligned} V_H(\phi_1, \phi_2, T) = & E'_{vac} + \frac{1}{2}c_1(T^2 - T_{c1}^2)(\phi_1^\dagger\phi_1) + \frac{1}{2}c_2(T^2 - T_{c2}^2)(\phi_2^\dagger\phi_2) \\ & + \lambda'_1(\phi_1^\dagger\phi_1)^2 + \lambda'_2(\phi_2^\dagger\phi_2)^2 + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2*}(\phi_2^\dagger\phi_1) \\ & + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2] \\ & + (\phi_1^\dagger\phi_1)[\lambda_6(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_2^\dagger\phi_1)] + (\phi_2^\dagger\phi_2)[\lambda_7(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_1)] \\ & + \text{higher order terms,} \end{aligned} \quad (28)$$

In this case corresponding to the two different vev's of the fields ϕ_1 and ϕ_2 , there would be occurring two different critical temperatures $T_{c1} = \sqrt{-\frac{2\rho_1\Lambda_1 m_{11}^2}{c_1}}$ and $T_{c2} = \sqrt{-\frac{2\rho_2\Lambda_2 m_{22}^2}{c_2}}$ respectively. In this analysis we require that the phase transitions in doublet ϕ_2 occur after phase transitions in ϕ_1 , thus $T_{c1} > T_{c2}$.

There are three possible situations depending upon the value of temperature,

$T > T_{c_1}$:

For $T > T_{c_1}$ the symmetry is not broken in either field and the phase transitions has not occurred. Thus, in this situation minima of both fields lies at zero given by eq. (20). All fields are massless in this temperature range i.e. $T \geq T_{c_1}$. Since, symmetry is not broken in this limit all the fermions and bosons are also massless.

$T_{c_2} < T < T_{c_1}$:

This is the most interesting situation as the symmetry is retained in field ϕ_2 and is broken in ϕ_1 . In this situation due to phase transitions in ϕ_1 all fields (fermions and all bosons) gets masses, but since the phase transitions in field ϕ_2 has not occurred so it still is a possible candidate for dark energy field. This situation is also interesting because this article assumes that the present observed Universe happens to occur in this domain. The expansion of ϕ_1 around its minimum ν_1 leads to exactly the same phenomenology as that of the SM.

$T < T_{c_2}$:

When $T < T_{c_2}$ symmetry has got broken spontaneously in both fields ϕ_1 and ϕ_2 at temperatures T_{c_1} and T_{c_2} respectively. In this situation, both fields gets perturbed around their true minima. This perturbation give masses to all the mixed fields of the doublets ϕ_1 and ϕ_2 (except the CP odd and charged field of angle $\beta = \tan^{-1} \frac{\nu_2}{\nu_1}$ rotated new Higgs doublet H_1 which are the Goldstone bosons). This should be expected to occur some time in the future considering $m_{22}^2 \neq 0$.

Constraint on parameters from phase transitions

If we require that the phase transitions in the doublet ϕ_1 (which is acting now to be SM Higgs doublet) occur at the same temperature as in SM Higgs doublet, we can constrain the parameters of the potential. Note that this condition is not necessary but an interesting one

which can probe this model (one can also relax this condition). The critical temperature at which phase transitions occur in SM is [23],

$$T_c^2 = \frac{4 \lambda \nu^2}{2\lambda + \frac{3}{4}g^2 + \frac{1}{4}g'^2}$$

where $\lambda = 0.1305$ is the quartic coupling of Higgs doublet in SM, $g = 0.6376$ is the Higgs doublet coupling with SU(2) gauge group and $g' = 0.3441$ is the Higgs doublet coupling with U(1) gauge group. With the values of parameters given above we get $T_c = 230.3186\text{GeV}$. Setting $T_{c_1} = T_c$ gives,

$$\sqrt{\frac{-2 \rho_1 \Lambda_1 m_{11}^2}{\frac{6\lambda'_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g_1^2 + g_1'^2}{32} + \frac{g_t^2 + g_b^2}{8}}} = \sqrt{\frac{4 \lambda \nu^2}{2\lambda + \frac{3}{4}g^2 + \frac{1}{4}g'^2}},$$

taking square and using equation (22) in,

$$\frac{2 \lambda'_1 \nu_1^2}{\frac{6\lambda'_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g_1^2 + g_1'^2}{32} + \frac{g_t^2 + g_b^2}{8}} = \frac{4 \lambda \nu^2}{2\lambda + \frac{3}{4}g^2 + \frac{1}{4}g'^2},$$

here $\nu^2 = \nu_1^2 = 246 \text{ GeV}^2$, λ_i 's are the parameters of inert doublet model potential, g_1 and g'_1 are the couplings of doublet ϕ_1 with the SU(2) and U(1) gauge group respectively. Simplification of the above equation with the numerical values lead to

$$2\lambda_3 + \lambda_4 = 0.7869, \quad (29)$$

Here, in deriving the eq. (29), $\lambda'_1 = 0.1305$, $g_1 = 0.6376$, $g'_1 = 0.3441$, $g_t = 1$ and $g_b = 0.03$ have been used.

IV. HIGGS FIELD(S) AS DARK ENERGY FIELD(S)

The second Friedmann eq. (2) tells that accelerated expansion will occur when $\omega_{\text{eff}} < -\frac{1}{3}$. For the field ϕ_2 to be the dark energy field, it must bring $\omega_{\text{eff}} < -\frac{1}{3}$ in the history of Universe (in fact just now $Z \approx 0.37$ with $\Omega_m = 0.3$ and $\Omega_{\text{DE}} = 0.7$) which tells that the symmetry

in this field should not be broken. For this purpose, we need to solve the Euler Lagrange equations in some background, which are

$$\partial_\mu \left(\frac{\partial(\sqrt{-g}\mathcal{L}_{Higgs})}{\partial(\partial_\mu\psi_i)} \right) - \frac{\partial(\sqrt{-g}\mathcal{L}_{Higgs})}{\partial\psi_i} = 0, \quad (30)$$

where ψ_i are different fields of doublets ϕ_1 and ϕ_2 .

The solution of Euler Lagrange equations of motion in FRW Universe ($\sqrt{-g} = a(t)^3$) for the fields ϕ_2^\pm , η_2 and χ_2 given above is

$$\begin{aligned} \ddot{\eta}_2 + 3\frac{\dot{a}}{a}\dot{\eta}_2 + \frac{1}{2} \left[\eta_2 \left((\lambda_3 + \lambda_4 + \lambda_5) \nu^2 + 2m_{22}^2 \Lambda_2 \rho_2 \exp \left(\frac{1}{2} m_{22}^2 \Lambda_2 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2}) \right) \right. \right. \\ \left. \left. + \lambda_2 \Lambda_4 \rho_4 \chi_2^2 \exp \left(\frac{1}{8} \lambda_2 \Lambda_4 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2})^2 \right) + 2\lambda_2 \Lambda_4 \rho_4 \phi_2^{c2} \exp \left(\frac{1}{8} \lambda_2 \Lambda_4 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2})^2 \right) \right) \right. \\ \left. \left. + \lambda_2 \Lambda_4 \rho_4 \eta_2^3 \exp \left(\frac{1}{8} \lambda_2 \Lambda_4 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2})^2 \right) \right] = 0 \end{aligned} \quad (31)$$

$$\begin{aligned} \ddot{\chi}_2 + 3\frac{\dot{a}}{a}\dot{\chi}_2 + \frac{1}{2} \left(\chi_2 \left((\lambda_3 + \lambda_4 - \lambda_5) \nu^2 + 2m_{22}^2 \Lambda_2 \rho_2 \exp \left(\frac{1}{2} m_{22}^2 \Lambda_2 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2}) \right) \right. \right. \\ \left. \left. + \lambda_2 \Lambda_4 \rho_4 \eta_2^2 \exp \left(\frac{1}{8} \lambda_2 \Lambda_4 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2})^2 \right) + 2\lambda_2 \Lambda_4 \rho_4 \phi_2^{c2} \exp \left(\frac{1}{8} \lambda_2 \Lambda_4 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2})^2 \right) \right) \right. \\ \left. \left. + \lambda_2 \Lambda_4 \rho_4 \chi_2^3 \exp \left(\frac{1}{8} \lambda_2 \Lambda_4 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2})^2 \right) \right) = 0 \end{aligned} \quad (32)$$

$$\begin{aligned} \ddot{\phi}_2^c + 3\frac{a'}{a}\dot{\phi}_2^c + \frac{1}{2} \left(\lambda_3 \nu^2 + 2m_{22}^2 \Lambda_2 \rho_2 \exp \left(\frac{1}{2} m_{22}^2 \Lambda_2 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2}) \right) \right. \\ \left. \left. + \lambda_2 \Lambda_4 \rho_4 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2}) \exp \left(\frac{1}{8} \lambda_2 \Lambda_4 (\chi_2^2 + \eta_2^2 + 2\phi_2^{c2})^2 \right) \right) \phi_2^c = 0 \end{aligned} \quad (33)$$

To avoid the confusion between positively and negatively charged Higgs boson, ‘c’ instead of + or – has been used.

The energy density and pressure after expansion of 2HDM Higgs Lagrangian for physical

fields is

$$\begin{aligned}
\rho_{Higgs}/P_{Higgs} = & \frac{1}{2}\dot{\eta}_2^2 + \frac{1}{2}\dot{\chi}_2^2 + \dot{\phi}_2^{c2} \pm \left[\frac{1}{4}(\lambda_3 + \lambda_4 + \lambda_5)\nu^2\eta_2^2 + \frac{1}{4}(\lambda_3 + \lambda_4 - \lambda_5)\nu^2\chi_2^2 \right. \\
& + \frac{1}{2}\lambda_3\nu^2\phi_2^{c2} + \rho_2 \exp\left(\frac{1}{2}m_{22}^2\Lambda_2\chi_2^2 + \frac{1}{2}m_{22}^2\Lambda_2\eta_2^2 + m_{22}^2\Lambda_2\phi_2^{c2}\right) \\
& + \rho_4 \exp\left(\frac{1}{8}\lambda_2\Lambda_4\chi_2^4 + \frac{1}{8}\lambda_2\Lambda_4\eta_2^4 + \frac{1}{2}\lambda_2\Lambda_4\phi_2^{c4} + \frac{1}{4}\lambda_2\Lambda_4\chi_2^2\eta_2^2 \right. \\
& \left. \left. + \frac{1}{2}\lambda_2\Lambda_4\chi_2^2\phi_2^{c2} + \frac{1}{2}\lambda_2\Lambda_4\eta_2^2\phi_2^{c2}\right) \right], \tag{34}
\end{aligned}$$

For the cosmological evolution of the fields η_2 , χ_2 and ϕ_2^c , the above equation of motion are solved with the Friedmann equations numerically in the flat Universe ($\kappa = 0$). The initial conditions used are $\eta_{2ini} = M_P$, $\chi_{2ini} = M_P$, $\phi_{2ini}^c = 0$, $\dot{\eta}_{2ini} = 0$, $\dot{\chi}_{2ini} = 0$ and $\dot{\phi}_{2ini}^c = 0$.

Note that after imposing the Z_2 symmetry there are five parameters which determine the masses of Higgs fields. With $m_{\eta_1} = m_{H_{SM}} = 125.7\text{GeV}$ and constraint from phase transitions, we have only two equations to determine parameters values[24]. The Higgs fields masses in this analysis were calculated using eq. (25). After imposing $m_{\eta_1} = m_{H_{SM}} = 125.7\text{GeV}$ and constraint from phase transitions eq. (29) some the arbitrary choice of parameters was taken[25] since there were only two equations to determine five unknowns.

Masses of Higgs bosons in the analysis are taken to be

$$m_{\eta_2} = 6.925 \times 10^{-61}\text{GeV}, \quad m_{\chi_2} = 6.925 \times 10^{-61}\text{GeV}, \quad m_{\phi_2^\pm} = 154.305\text{GeV},$$

just to set the evolution of the energy densities as observed. But too small masses of lightest Higgs fields were required to induce high damping in the field's oscillations which is required to ensure that ω_{eff} get less than $-1/3$ only once on the cosmological scale and whenever it gets it never gets value greater than $-1/3$. Since the vacuum energy of the Higgs is too small $\mathcal{O}(10^{-121}E_P/L_P^3)$, it asks for very small values of parameters to achieve over-damped oscillations. Thus, the fine tuning problem related to the smallness of field mass in scalar field models also exists in our model too.

To make the Universe evolving in the vacuum states given by Inert vacuum II, $m_{22}^2 = 0$ has been chosen which implies $T_{c_2} = 0$. In this situation, phase transitions in the second doublet will never occur (no appearance of odd forces and particles would be observed) and the Universe will keep on accelerating forever.

The solution of the eqs. (31, 32, 33) along with Friedmann equations is shown below

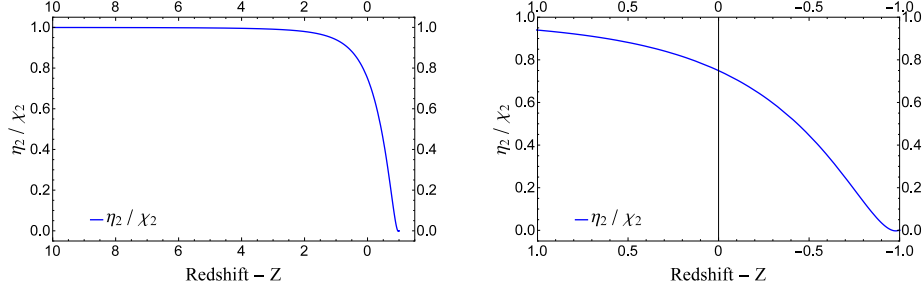


FIG. 1: Higgs field as a function of redshift.

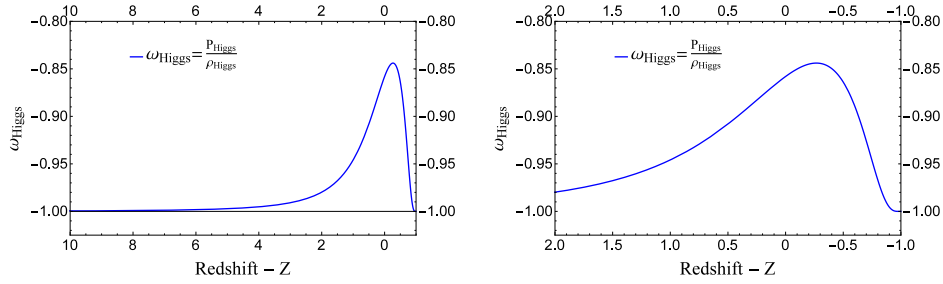


FIG. 2: Equation of state ω_{Higgs} for Higgs fields, as seen it starts with -1 then evolves towards quintessence regime after large enough time it comes back at -1 .

During the initial stages $Z \approx 3750$, the evolution of the Higgs fields η_2 , χ_2 and ϕ_2^\pm are frozen by the expansion shown in fig. (1) and acts as a negligibly small vacuum energy component with $\omega = -1$. As time proceeds, Higgs fields begins to evolve towards minimum of potential, the energy density in the Higgs fields start to dominate cosmologically (on Hubble scale). During the evolution equation of state parameter ω_{Higgs} starts to increase and becomes > -1 as shown in fig. (2). In the very late (future) Universe ($Z \leq 0$), the fields comes to rest at the minimum of the potential and a period with $\omega = -1$ is reached to give an exponentially accelerating Universe. Since, $\omega_{Higgs} \not\approx -1/3$ at any time in evolution, after ω_{eff} gets less than $-1/3$ in the evolution, an ever accelerating Universe is obtained in this model.

As discussed before, Higgs field stability is needed to get an ever accelerating expanding

Universe. This was obtained by imposing the discrete Z_2 symmetry. The lightest Higgs fields η_2 and χ_2 in this case do not decay into any other Higgs field (since these fields are lighter than SM like and charged Higgs) and also into fermions (since they do not couple to them). Thus, we have a model of ever accelerating expanding Universe.

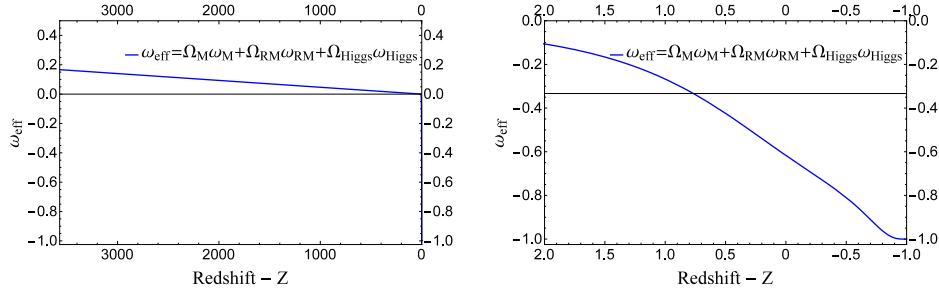


FIG. 3: Effective equation of state $\omega_{eff} = \Omega_{DE}\omega_{DE} + \Omega_R\omega_R + \Omega_M\omega_M$.

The ω_{eff} in the fig. (3) starts from ≈ 0.167 (set by initial conditions $\Omega_{\text{Higgs}_{int}} = 0$ and $\Omega_{NR_{int}} = \Omega_{R_{int}} = 0.5$; NR: non-relativistic matter and R: relativistic matter) and decreases as the relativistic matter's energy density decreases. This has been shown in fig. (3) for ω_{eff} and in fig. (4) for energy densities and comes down at 0. This and before is the time period when non-relativistic matter dominates and expanding Universe decelerates with highest rate ($\Omega_{NR} \approx 1$) as non-relativistic matter domination pulls every thing inwards more than outwards Higgs negative pressure. After that ω_{eff} starts to decrease as the non-relativistic energy density decreases and Higgs relic energy density increases as shown in fig. (6) and (7) this time and afterwards Higgs negative pressure starts to dominates for forever (because second phase transition never occurs in our model) and ω_{eff} eventually settles down at -1 .

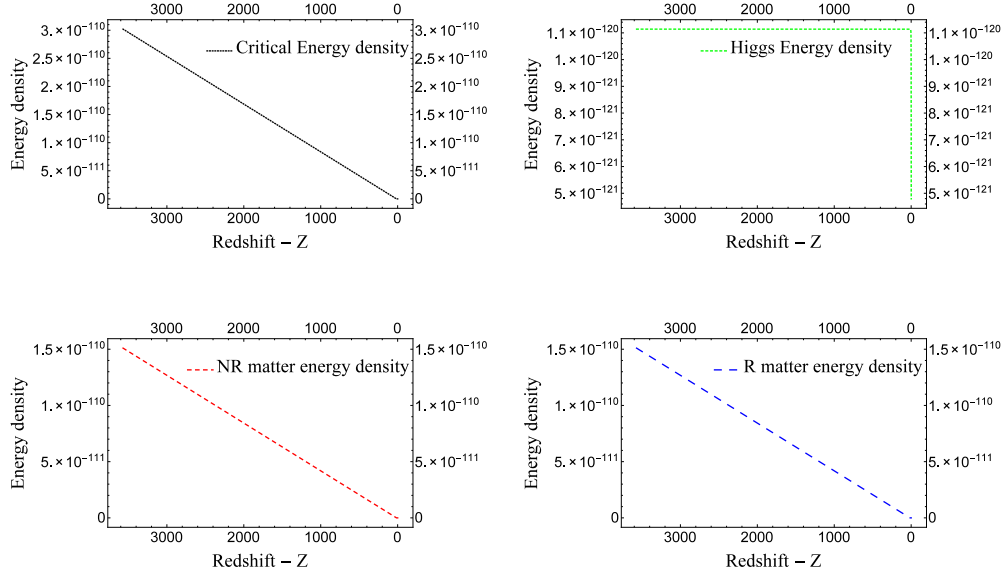


FIG. 4: Energy densities of different components as a function of redshift.

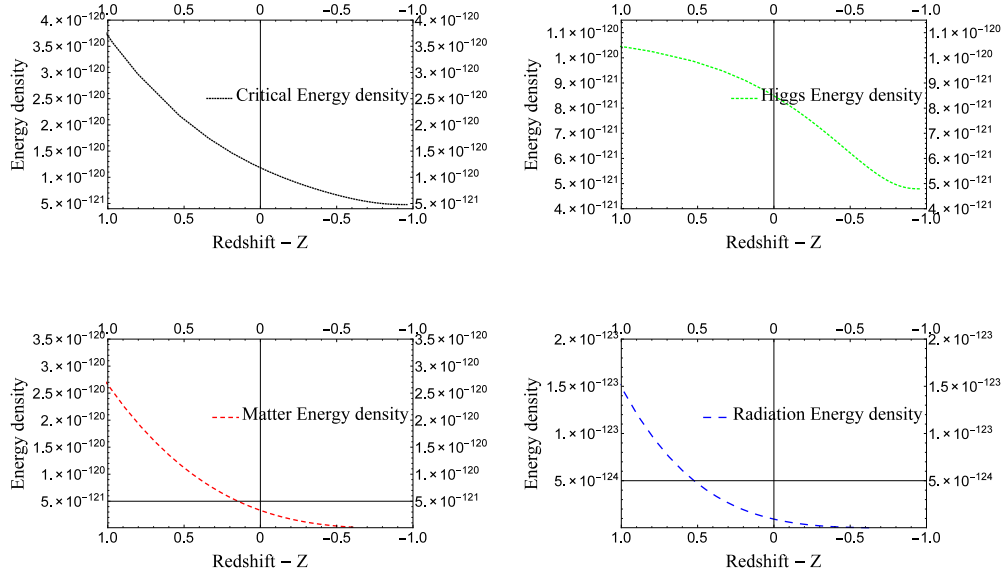


FIG. 5: Energy densities of different components as a function of redshift.

Figure (4) and (5) show the evolution of different components of the Universe as a function of redshift Z . In Fig. (4) we can see the evolution of different components from $Z \approx 3750$

to $Z \approx -1$ while fig. (5) shows the evolution from $Z = 1$ to $Z \approx -1$. From fig. (5), we note that the energy density decreases as a function of a^{-3} (a^{-4}) for non-relativistic matter (relativistic matter) while from fig. (4) it can be seen that for high redshifts the energy density for Higgs fields remains approximately constant. However, for low redshifts (from fig. (5)) the Higgs energy density decreases to settle down at their minimum value (E_{vac} given by eq. (9)) as the fields come to rest at low redshifts (shown in fig. (1)).

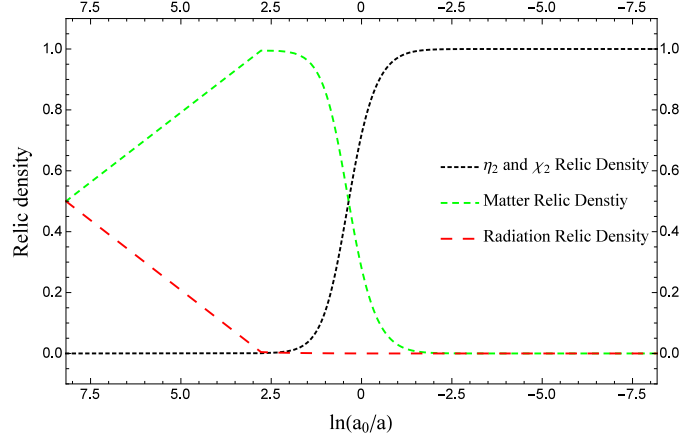


FIG. 6: Relic densities of different components as a function of $\ln[a_0/a]$.

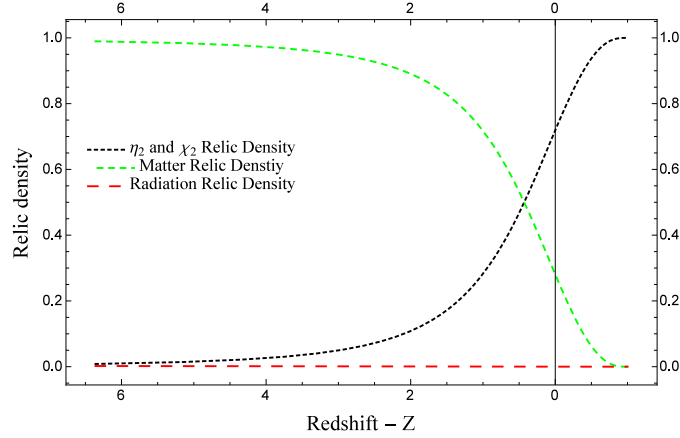


FIG. 7: Relic densities of different components as a function of redshift.

Figure (6) is plotted for the evolution of the relic density of the non-relativistic matter, relativistic matter and the dark energy Higgs fields against $\ln(a_0/a(t))$, starting from the

non-relativistic and relativistic matter equality; where t is time. The figure shows that only now the dark energy Higgs has started to dominate in the critical density while in the past non-relativistic matter was dominating. Figure (7) is plotted for the evolution of the relic density of the non-relativistic matter, relativistic matter and the dark energy Higgs fields against the redshift Z starting from the non-relativistic and relativistic matter equality to $Z \approx -1$; where t is time. The figure shows that for $Z \leq 6$ the dark energy Higgs has started to become prominent. At $Z \approx 0.3235$ the non-relativistic matter and relativistic matter energy densities are equal. For $Z > 0.3235$ non-relativistic matter is dominating and for $Z < 0.3235$ dark energy Higgs started to dominate causing the accelerated expansion of the Universe.

It is quite clear from the plots that component Higgs fields of second Higgs doublet are a possible candidate for the dark energy if the present Universe happens to be evolving in the vacuum states described by inert doublet model given by the vacuum solution eq. (22). One needs to know that the initial conditions for charged field were taken in accordance with the observation that dark energy (vacuum) is not charged.

V. DECAY(S) OF HIGGS FIELD(S)

At tree level the doublet ϕ_2 in inert doublet model discussed above whose component fields are the candidate for dark energy is only coupled to the doublet ϕ_1 . The doublet ϕ_1 acts in an identical way as SM Higgs doublet.

The interaction Lagrangian of Higgs fields of doublet ϕ_2 with Higgs fields of doublet ϕ_1 and

gauge bosons (extracted from eq. (5)) is

$$\begin{aligned}
\mathcal{L}_I = & \frac{1}{2}\nu_1(\lambda_3 + \lambda_4 - \lambda_5)\chi_2^2\eta_1 + \frac{1}{2}\nu_1(\lambda_3 + \lambda_4 + \lambda_5)\eta_2^2\eta_1 + \lambda_3\nu_1\eta_1\phi_2^+\phi_2^- \\
& + \frac{1}{4}(\lambda_3 + \lambda_4 - \lambda_5)\chi_2^2\eta_1^2 + \frac{1}{4}(\lambda_3 + \lambda_4 + \lambda_5)\eta_2^2\eta_1^2 + \frac{1}{2}\lambda_3\eta_1^2\phi_2^+\phi_2^- \\
& + \frac{g_2^2g_2'^2}{(g_2'^2 + g_2^2)}\phi_2^+\phi_2^-A_\mu^2 + \frac{(g_2'^2 + g_2^2)}{8}\eta_2^2Z_\mu^2 + \frac{(g_2'^2 + g_2^2)}{8}\chi_2^2Z_\mu^2 + \frac{(g_2^2 - g_2'^2)^2}{4(g_2'^2 + g_2^2)}\phi_2^+\phi_2^-Z_\mu^2 \\
& + \frac{g_2g_2'(g_2^2 - g_2'^2)}{(g_2'^2 + g_2^2)}\phi_2^+\phi_2^-A_\mu Z_\mu + \frac{g_2^2}{4}W_\mu^-W_\mu^+(\eta_2^2 + \chi_2^2 + 2\phi_2^+\phi_2^-) \\
& + \frac{g_2^2g_2'}{2\sqrt{(g_2'^2 + g_2^2)}}\eta_2A_\mu(\phi_2^+W_\mu^- + \phi_2^-W_\mu^+) + i\frac{g_2^2g_2'}{2\sqrt{(g_2'^2 + g_2^2)}}\chi_2A_\mu(\phi_2^-W_\mu^+ - \phi_2^+W_\mu^-) \\
& - \frac{g_2g_2'^2}{2\sqrt{(g_2'^2 + g_2^2)}}\eta_2Z_\mu(\phi_2^+W_\mu^- + \phi_2^-W_\mu^+) - i\frac{g_2g_2'^2}{2\sqrt{(g_2'^2 + g_2^2)}}\chi_2Z_\mu(\phi_2^-W_\mu^+ - \phi_2^+W_\mu^-).
\end{aligned} \tag{35}$$

To suppress the interaction of Higgs fields η_2 , χ_2 and ϕ_2^\pm with the gauge bosons the idea is that the SU(2) doublet ϕ_2 is very weakly (and different than ϕ_1) coupled with the gauge bosons, thus here $g_2 \lll g_1$ and $g_2' \lll g_1'$. Thus, the decay modes that include the Higgs fields of the second Higgs SU(2) doublet are negligible as compared to the modes that include the SM Higgs.

The Higgs decay to a pair of (Higgs) scalar while considering only the on-shell width, the decay width is given by [19]

$$\Gamma(H_i \longrightarrow H_j H_k) = (2 - \delta_{jk})m_{H_i} \frac{|C_{H_i H_j H_k}|^2}{32\pi} \sqrt{f \left(1, \frac{m_{H_j}^2}{m_{H_i}^2}, \frac{m_{H_k}^2}{m_{H_i}^2} \right)}, \tag{36}$$

where

$$f \left(1, \frac{m_{H_j}^2}{m_{H_i}^2}, \frac{m_{H_k}^2}{m_{H_i}^2} \right) = \left(1 - \frac{m_{H_j}^2}{m_{H_i}^2} - \frac{m_{H_k}^2}{m_{H_i}^2} \right)^2 - 4 \frac{m_{H_j}^2}{m_{H_i}^2} \frac{m_{H_k}^2}{m_{H_i}^2}.$$

$C_{H_i H_j H_k}$ is the coupling of different Higgs bosons H_i , H_j and H_k .

From eq. (35) $C_{\eta_1 \chi_2 \chi_2} = \frac{1}{2}\nu_1(\lambda_3 + \lambda_4 - \lambda_5) = \frac{m_{\chi_2}^2}{\nu_1}$, $C_{\eta_1 \eta_2 \eta_2} = \frac{1}{2}\nu_1(\lambda_3 + \lambda_4 + \lambda_5) = \frac{m_{\eta_2}^2}{\nu_1}$ and $C_{\eta_1 \phi_2^+ \phi_2^-} = \lambda_3 \nu_1 = 2\frac{m_{\phi_2^S}^2}{\nu_1}$. The decay rate of η_1 to $\eta_2 \eta_2$, $\chi_2 \chi_2$, $\phi_2^- \phi_2^+$ for the masses used in

the cosmological evolution determination in the previous section are,

$$\Gamma(\eta_1 \longrightarrow \chi_2 \chi_2) = 1.53794 \times 10^{-6} \text{ GeV}$$

$$\Gamma(\eta_1 \longrightarrow \eta_2 \eta_2) = 1.53794 \times 10^{-6} \text{ GeV}$$

$$\Gamma(\eta_1 \longrightarrow \phi_2^+ \phi_2^-) = 0. \text{ GeV}$$

Also the decay rates for SM like Higgs boson is given in the graph below as a function of masses m_{η_2} , m_{χ_2} and $m_{\phi_2^\pm}$,

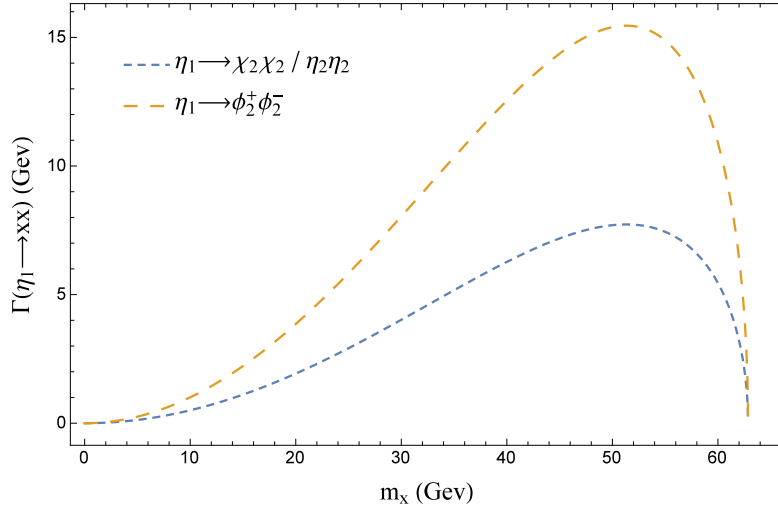


FIG. 8: Decay rates for $\eta_1 \longrightarrow xx$

From the above plot, we see that the decay rate for SM like Higgs to other Higgs pairs is peaked at $m_x = 51.3168 \text{ GeV}$. For $m_x < 51.3168 \text{ GeV}$ the decay rate increases from zero to $\Gamma = 15.4557 \text{ GeV}$ and $\Gamma = 7.72786 \text{ GeV}$ for charged pair and neutral pair respectively, after that it decreases sharply and goes to zero at $m_x = 62.85 \text{ GeV}$.

It should also be mentioned that total decay width of SM like Higgs boson here is well within bounds of mass resolution $\approx 12 \times 10^{-3} \text{ GeV}$ of LHC [26] (this is only the current results and the uncertainty in the measurements is high [26] which is expected to be refined [could come to SM expected value as more data is collected]) for the masses used in the cosmological evolution of the Higgs fields. One should also mention that the SM prediction of total decay width for Higgs boson is $4.21 \times 10^{-3} \text{ GeV}$ with mass 126 GeV [27] and is $4.07 \times 10^{-3} \text{ GeV}$ with

mass 125GeV [26]. In this model, we get three more decay channels of SM like Higgs which are to the other Higgs bosons pair.

It should also be mentioned that the dark energy density does not modify as a consequence of the SM-like Higgs decay into dark energy Higgs pair for the masses of the Higgs fields taken here. Had we been able to achieve accelerated expansion with the mass of the second Higgs $\geq 125\text{GeV}$ (which we could not because of fine tuning problem) the dark energy density would change significantly at redshift corresponding to the equilibrium temperature (freeze out temperature of dark energy Higgs) $T_{eq} = m_{H_2} - 2m_{H_{SM}}$. In that case, we could have a Universe whose different regions would accelerate, decelerate or contract differently – a case usually called bubble Universes, multi-verses or parallel Universes. These decays do not imply any additional fine tuning.

VI. CONCLUSIONS

Scalar fields are among the possible candidate for the observed accelerated expansion of the Universe. In this article, I argued that the Particle Physics developed so far must have something in/beyond SM which will explain the observed accelerated expansion of the Universe and hence will serve as the dark energy candidate. This is also necessary for the unification of matter and vacuum energy. This unification is necessary in my opinion. This article took specifically this approach that dark energy actually is some form of scalar field(s) which is(are) present in the so called inert doublet model.

I have found that if the present Universe is described by the vacuum given by an inert doublet model then the component scalar fields of the inert doublet ϕ_2 can be one possible candidate for the dark energy. Since the present contribution of the dark energy to the critical energy density is about 0.7, this value is obtained by taking the masses of CP-even field(s) very low. The most important thing is that with the initial conditions set the mass of the charged (ϕ_2^c) field become arbitrary if we release the parametric constraint given by eq. (29) which is obtained using phase transitions bound, this model will fit for any value of mass of ϕ_2^c . One needs to keep in mind that the values of masses were chosen arbitrary just to get dark energy relic density ≈ 0.7 , changing the values of masses mildly does not

changes the relic density much. We could also have some other masses set which would give the same evolution as shown below for above masses set but there is some bound on the parameters of the potential to get high damping in the fields (which restricts the masses to remain in certain order of energy).

As discussed above even though the masses were chosen arbitrary to get $\Omega_{Higgs} \approx 0.7$, to avoid the oscillations in the fields we have to take $\lambda_3 + \lambda_4 + \lambda_5 \approx \mathcal{O}(10^{-125})$ and $\lambda_3 + \lambda_4 - \lambda_5 \approx \mathcal{O}(10^{-125})$ and thus the masses of second Higgs doublet very small to ensure that we get $\omega_{\text{eff}} = -1/3$ only once in the history of the Universe. The obtained results suggests to probe the low energy behavior at the Universe's current temperature ($T = 2.73^\circ\text{K}$) of the standard model and its extensions to obtain new information and possibility of dark energy candidacy in Particle Physics.

To suppress the interaction of the Higgs fields η_2 , χ_2 and ϕ_2^\pm with the gauge bosons W^\pm , Z and $A(\mu)$, we imposed the condition that the SU(2) doublet ϕ_2 is very weakly (and different than ϕ_1) coupled with the gauge bosons. Thus here $g_2 \lll g_1$ and $g'_2 \lll g'_1$. This allowed us to conclude that the decay modes that include the Higgs fields η_2 , χ_2 and ϕ_2^\pm are negligible as compared to the modes that include the SM Higgs. The couplings g_1 and g'_1 have the values of the SM Higgs gauge couplings in the SM but the values of g_2 and g'_2 must be set using the results from LHC or any other accelerator.

One thing that remains important to check in all extensions of the SM is whether the Higgs potential contains the vacuum instability or not? If it does, then does it make the vacuum instability more worse as compared to the SM or not? The answer to the question for our model is that although it contains the vacuum instability, due to the coupling of the second Higgs with the SM Higgs is $\mathcal{O}(10^{-126})$, it will not affect the RGEs running of the SM Higgs. Thus will not make the vacuum instability worse. We expect the vacuum instability to occur at approximately the same scale as it occurs in the SM.

When T becomes less than T_{c_2} , symmetry in both fields is broken and all mixed fields of both doublets get massive (except Goldstone bosons). In this situation, one neutral Higgs can be made to acts in the same way as SM Higgs does (i.e. giving masses to other particles and having mass 125.7GeV) while other fields can be made to act as dark energy field(s) by setting the Yukawa interaction to cancel the other fields effect on fermions. This should be

investigated in future.

When describing a model for accelerated expansion of the Universe, it becomes essential to compare it with the Λ CDM model. In comparing our model with the Λ CDM, we note that on the cosmological scale our model is completely different from the standard Λ CDM and also very predictive. In our model ω_{Higgs} is not constant over redshift which has been shown in fig. (2). From fig. (2), we see that the model gives $\omega_{Higgs} \approx -0.858$ at $Z = 0$. Thus our model can be distinguished from the Λ CDM via the variation of ω_{Higgs} from -1 on the cosmological scale. The deviation of ω_{Higgs} at low redshift $2 < Z < 0$ is given in fig. (2). In the plot, we see that while we started from $\omega_{Higgs} = -1$ at $Z \approx 3750$, around $Z \approx 6$ ω_{Higgs} start to get greater than -1 . At redshift $Z = 0$ the deviation of ω_{Higgs} from -1 we obtain in our model for explicit values of parameters using MATHEMATICA software is $0.1419828 \pm 3.2433649325 \times 10^{-8}$. In concluding, since we get $\omega_{eff} < -1/3$ after solving the Euler Lagrange equations numerically, the proposed Higgs field could cause the current observed accelerated expansion.

It is also worth concluding that because of the type of Yukawa interaction the second phase transition (the case of $m_{22}^2 \neq 0$) will not change fermions phenomenology but due to the $SU(2) \otimes U(1)$ gauge invariance gauge bosons phenomenology will get changed (in general). Setting the coupling constants g_2 and g'_2 might end up not changing gauge bosons phenomenology subsequently. This should be investigated in future theoretically.

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$$\begin{aligned}
\mathcal{L}_{gf}^{SM} = & -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& + \bar{\psi}_L^i \gamma^\mu \nabla_\mu^{EW} \psi_L^i + \bar{\psi}_R^i \gamma^\mu \nabla_\mu^{EW} \psi_R^i \\
& + \bar{\chi}_L^i \gamma^\mu \nabla_\mu^{SM} \chi_L^i + \bar{U}_R^i \gamma^\mu \nabla_\mu^{SM} U_R^i + \bar{D}_R^i \gamma^\mu \nabla_\mu^{SM} D_R^i .
\end{aligned} \tag{37}$$

[16] .

$$\mathcal{L}_Y = -Y_{ij}^u \bar{\chi}_L^i \tilde{\phi}_1 U_R^j - Y_{ij}^d \bar{\chi}_L^i \phi_1 D_R^j - Y_{ij}^e \bar{\psi}_L^i \phi_1 \psi_R^j - h.c. \tag{38}$$

here ψ_L^i are left handed leptons doublets, ψ_R^i are right handed leptons singlets, χ_L^i are left

handed quark doublets, U_R^i and D_R^i are the right handed quark singlets. i runs from 1-3. ϕ_1 is the SM like Higgs doublet.

- [17] In the SM one scalar isodoublet with hypercharge $Y = 1$ is sufficient to make the theory complete and gauge invariant and hence in SM

$$\mathcal{L}_{Higgs} = T_H - V_H = (D_\mu \phi)^\dagger (D^\mu \phi) - \left(-\frac{\mu^2}{2!} \phi^2 + \frac{\lambda}{4!} \phi^4\right) \quad (39)$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{and} \quad \tilde{\phi} = i\sigma^2 \bar{\phi} = \begin{pmatrix} \bar{\phi}^0 \\ -\bar{\phi}^+ \end{pmatrix}.$$

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